# Logical Groundwork

Principle 1

Geometry deals with **Entities**, **Qualities**, and **Places**

Principle 2

We could construct any entity or quality by collecting a finite collection of qualities, this collection we will call the entity’s **Identity**. The different qualities we will call **Identifiers**. Notate:

Principle 3

There are **Kinds** of entities and of qualities. Members of the same kind have identities with the same number, order and types of qualities. The order by itself is arbitrary as long as it is consistent.

Principle 4

Assuming relations, we assume a binary relation of **Identicality** or being the same upon the qualities and the places. It is reflexive symmetric and transitive. Notate:

**Define:** If two things are identical, any proposition that is true for one is also true of the other, them filling the same rule in the proposition.

Principle 5

Any entity has a single place. Notate:

Principle 6

Two entities that have their whole identity the same, to say every quality of the first is identical to the corresponding quality are themselves identical. Regardless of place.   
[Euc b1.cn.4]

Principle 7

For an entity that is constructed by properties of the type . We could always reconstruct any specific from e and from the place of that in the order among the s. Notate:

Principle 8

As we assume the natural numbers, excluding the zero, certain qualities have numeric values, these qualities are called **Measures.**

# Common Notions and Theorems about Measures

1. Identical measures are called equal, this is also notated by
2. There are operations between two measures such that it possible that they’ll produce a third measure, it being equal to the first two with the operation
3. We assume an addition operation which is associative and commutative. Notate:

Two measures could be added to each other, thus always producing a third measure. Notate:

1. Let there be a subtraction operation. Notate:

Two Measures could sometimes be subtracted from each other producing a third. Notate:

**Define:** For two measures, exist a yet third measure such that the second added to the third produce the first, if and only if the second subtracted from the first produces the third.

1. Let there be a bigger than relation such that one measure could be bigger than another. Notate:

**Define**: for two measures, exist a mesure identical to the second subtracted from the first if and only if the first is bigger than the second

[similar to but don’t completes Euc b1.cn.5]

1. For any three measures: subtracting the second from the first and to that add the third produce an equal measure the the one produced by adding the third to the first and from that subtract the second. Notate:

From that follows that even if , we could know that .

1. A measure that is the product of subtracting of any measure from itself doesn’t exist in our system. Notate
2. For any two measure at least one of the following is true: they equal each other, the first is bigger than the second or the second is bigger than the first. Notate:

# Collaleries about measures:

1. If two mesure are equal, and the second is equal to a still third, the first is equal to the third. Notate:

[Euc b1.cn.1]

**Proof:**

Follows immedietly from the properties of identity, we just exchange for in

1. If a mesure if a product of the subtraction of a yet second measure from a third, then subrtracting the first measure from the third produces the second. Notate:

**Proof:**

# Theorems about Measures and their operations

1. If two mesures are equal, the mesures that are produced from the same by adding or subtracting to each a yet third measure, are (if existing) equal as well. Notate:

[Euc b1.cn.2,3]**Proof:**

Then from the definition of identity, the product of and of with the same operation and same measure has to be identical as well

Addition:

Subtraction:

1. For a mesure adding a second measure and than subtracting the same always produces the first measure.

Notate:

**Proof**

**Collalery** since we know from common notion 7 that

1. Adding a mesure to the product of subtraction (if existing) of yet two other measures is the same as adding the first of the two and than subtracting the second of the two.

**Proof:**

1. Subtracting from a mesure the product of addition of yet two other measures is the same as subtracting from it one of them and then the other notate:

**Proof**

The required symetry is obvious as

So now we know

1. Subtracting from a mesure the product of subtraction (if existing) of yet two other measures is the same as subtracting from it the first of the two and to that add the second of the two. Notate

**Proof:**

1. Addition of a single measure changes any measure, that is to say for any measure doesn’t exist a yet second measure such adding it to the first produces the first. Notate:

**Proof:**

From theorem 11

From common notion 7

# Theorems about Measures and ‘bigger than’

1. For two measures such that the first is bigger than the last is not bigger than the first.

**Proof:**

Since bigger than is not symetrical, a mirror notation will be introduced, and will be termed ‘smaller than’. Notate:

1. If a two measure are equal one is not bigger than the other, if one is bigger than the other, they are not equal. Notate

**Proof:**

which Theorem 16 state is not the case

From symetry of equality proving for bigger than also cover smaller than.

1. If one measure is bigger than a second measure, the mesures that are produced from the same by adding or subtracting to each a yet third measure, reatain (if existing) the relation of the product from the first being bigger than the product from the second. Notate:

**Proof:**

Addition:

From definition of larger than and subtraction

1. For any three mesures such that the first have one relation to the second and the second another relation to the third, if none of the relations are smaller than, and at least one of them is bigger than, than the first is bigger than the third.

**Proof:**

There are really three cases

As such

For i and ii

For iii

from the property of identity.

note from the property of addition.

# Book 1: Construction

## The point

**Definition 1**

a point is the entity with an empty identity. Notate:

Therfore any point is identical to any other.

**Definition 2**

a direction is the mode of difference in place between two points. It’s a quality. Notate:

An identity for a direction could be constructed from two not identical points.

We say that is to given direction of .

Any direction has an inverse direction.notate:

Such that

When talking about a pair of inverse direction we call them an orientation. Notate:

The properties of directions are:

1. Direction is anticommutative
2. It is transitive
3. And it is weakly backwardly transitive.

**Definition 3**

A directional value is a mesure given to a point relative to a certain direction. Notate:

An identity for such value could be constructed from a point and a direction, making also a second notation.

A property of directional value is that if one point is to that direction of another point, the dirval of the first is bigger than the dirval of the latter. Notate:

**Definition 3**

A *Distance* is the measure of difference of place between 2 points in their orientation. Notate:

An identity for distance could be constructed from two points.

The measure is calculated as such